Matter Universe: Message in the Mathematics

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Evidently the fact that the observable universe is dominated by matter at the expense of anti-matter is considered a major unsolved problem. Using the mathematical modeling based on the algebra $\mathbf{T} := \mathbf{C} \otimes \mathbf{H} \otimes \mathbf{O}$, an interpretation is developed that implies the existence of a matter universe, and an anti-matter universe, and more.

$\mathbf{T}^2$ Spinors: Particle Identifications

Bases for the real division algebras, $\mathbf{C}$, $\mathbf{H}$, $\mathbf{O}$ (complex algebra, quaternions, and octonions), are [1][2][3]:

$$\begin{array}{cl} \mathbf{C} & \{1,\ i\} \\ \mathbf{H} & \{q_0 = 1,\ q_k,\ k = 1,2,3\} \\ \mathbf{O} & \{e_0 = 1,\ e_a,\ a = 1,...,7\} \end{array}$$

The algebra

$$\mathbf{T} = \mathbf{C} \otimes \mathbf{H} \otimes \mathbf{O}$$

is $2 \times 4 \times 8 = 64$-dimensional. It is noncommutative, nonassociative, and nonalternative.

Although I consider it but a restricted model of reality, the basis of what I will do here is the 10-dimensional space-time model developed in [1], with mathematical expansion to be found in [2]. In this model, which accounts for a single family of quarks and leptons, and a corresponding antifamily, the foundation is the 128-dimensional hyperspinor space

$$\mathbf{T}^2$$

(the doubling of $\mathbf{T}$ in the spinor space is modeled on the notion that a Dirac spinor is a double Pauli spinor).

A Dirac spinor is acted upon by the Dirac algebra,

$$\mathbf{C}(4) \simeq \mathbf{P}(2),$$

where the Pauli algebra

$$\mathbf{P} \simeq \mathbf{C}(2) \simeq \mathbf{C} \otimes \mathbf{H}.$$

This is the complexification of the Clifford algebra if 1,3-spacetime. Likewise $\mathbf{T}^2$ is acted upon by the complexification of the Clifford algebra of 1,9-spacetime, represented by

$$\mathbf{T}_L(2),$$

where $\mathbf{T}_L$ is the algebra of left actions of $\mathbf{T}$ on itself, which in the octonion case, due to nonassociativity, requires the nesting of actions (as this has been covered ad nauseum in [1][2], I will proceed as though this is understood, although it is probably not).

If memory serves (and it serves less well every year), von Neumann and others [4], in their efforts to expand quantum theory from a foundation on $\mathbf{C}$ to one on $\mathbf{O}$, linked quantum observability with algebraic associativity, and unobservability with nonassociativity, thinking along these lines being forced by the nonassociativity of $\mathbf{O}$. This vague recollection partially motivates what follows.

In particular, the model building in [1][2] relies heavily on a resolution of the identity of

$$\mathbf{S} := \mathbf{C} \otimes \mathbf{O}$$

into a pair of orthogonal idempotents,

$$\rho_{\pm} = \frac{1}{2}(1 \pm ie_7).$$

With these $\mathbf{S}$ can be divided into 4 orthogonal subspaces:

$$\begin{aligned} \mathbf{S}_{++} &= \rho_+ \mathbf{S} \rho_+, \\ \mathbf{S}_{--} &= \rho_- \mathbf{S} \rho_-, \\ \mathbf{S}_{+-} &= \rho_+ \mathbf{S} \rho_-, \\ \mathbf{S}_{-+} &= \rho_- \mathbf{S} \rho_+. \end{aligned}$$

Both $\mathbf{S}_{++}$ and $\mathbf{S}_{--}$ are associative subalgebras of $\mathbf{S}$ isomorphic to $\mathbf{C}$. $\mathbf{S}_{+-}$ and $\mathbf{S}_{-+}$ are not subalgebras, but they are highly nonassociative (this nonassociativity implying $\mathbf{S}_{\pm\mp}^2 = \mathbf{S}_{\mp\pm}$ (you'd better check that - it's not relevant, but I never noticed that before - hmm)). Anyway, elements of the first two sets are linear (over $\mathbf{C}$) in the octonions $\{e_0 = 1,\ e_7\}$, and the second two sets linear over $\{e_p,\ p = 1, 2, 3, 4, 5, 6\}$.

An elegant representation of the Clifford algebra $\mathcal{CL}(1, 9)$ represented in $\mathbf{T}_L(2)$ that is aligned with the choice of the octonion unit e_7 to appear in $\rho_{\pm}$ arises from the following set of ten anti-commuting 1-vectors:

$$\beta,\ \ \gamma q_{Lk} e_{L7},\ k = 1, 2, 3,\ \ \gamma i e_{Lp},\ p = 1, ..., 6,$$

where

$$\epsilon = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ \alpha = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix},\ \beta = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix},\ \gamma = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix},$$

and as usual the subscripts "L" and "R" signify an action from the left or the right on $\mathbf{T}$. (So, for example,

$$\mathbf{S}_{+-} = \rho_+ \mathbf{S} \rho_- = \rho_{L+} \rho_{R-}[\mathbf{S}].)$$

However, our observable spacetime has 3 space dimensions, not 9. There are two (what I would call) canonical ways of reducing the 1-vectors of $\mathcal{CL}(1,9)$, a mix of observable and unobservable dimensions, to the 1-vectors of observable $\mathcal{CL}(1,3)$:

$$\rho_{L\pm}\ \{\beta,\ \ \gamma q_{Lk}e_{L7},\ k=1,2,3,\ \ \gamma ie_{Lp},\ p=1,...,6\}\rho_{L\pm}$$

$$=\ \{\beta,\ \ \gamma iq_{Lk},\ k=1,2,3\}\rho_{L\pm}.$$

These two collections of $\mathcal{CL}(1,3)$ 1-vectors act on half of the full spinor space $\mathbf{T}^2$. In particular, they act respectively on

$$\rho_{L\pm}[\mathbf{T}^2]=\rho_{\pm}\mathbf{T}^2,$$

where the underlying mathematics implies that these are, respectively, the matter and anti-matter halves of $\mathbf{T}^2$ ($\rho_+\mathbf{T}^2$ being a full family of lepton and quark Dirac spinors, and $\rho_-\mathbf{T}^2$ the corresponding anti-family) (see [1][2]).

It did not previously occur to me to interpret things in this way, but now I am willing to conclude that this implies that 1,3-spacetimes are inherently matter universes, or antimatter. That is, the observable (habitable, if you will) spacetime in which we reside must of necessity be a matter universe, with anti-matter arising from secondary interactions, or an anti-matter universe, and that both must exist. The question therefore arises, if there is just our matter universe, and a single separate anti-matter universe, do they evolve in tandem? That is, is there an anti-matter me presently typing an anti-matter version of this article on an anti-computer? Curiously, that anti-me doubtless thinks of himself as being composed of matter. He is wrong, of course. I am the matter me; he the anti-matter.

One concluding point: quarks, like the extra 6 dimensions of space in this model, are evidently not directly observable. And like the extra 6 dimensions of space, they owe their existence to the octonion units $e_p,\ p=1,2,3,4,5,6$. To reduce the spinor space $\mathbf{T}^2$ all the way to its observable lepton part (the anti-lepton part is similar) we need an extra ρ_+. Specifically,

$$\rho_{L\pm}\rho_{R\pm}[\mathbf{T}^2]=\rho_+\mathbf{T}^2\rho_+$$

is a lepton doublet, consisting of 2 Dirac spinors, one for the electron, one for its neutrino. (The particle identifications are not arbitrary. See particularly [2] for the mathematics behind that statement.) Interestingly, this further reduction does not result in any further reduction of the 1-vector space of our original Clifford algebra, $\mathcal{CL}(1,9)$. We're still left with a version of 1-vectors for $\mathcal{CL}(1,3)$. However, the story is different for the space of 2-vectors. Initially they form a representation of the 1,9-Lorentz Lie algebra, $so(1,9)$. After the initial reduction we get something more than $so(1,3)$:

$$\rho_{L+}so(1,9)\rho_{L+}=(so(1,3)\times so(6))\rho_{L+},$$

and after the second spinor reduction,

$$\rho_{R+}\rho_{L+}so(1,9)\rho_{L+}\rho_{R+}=(so(1,3)\times u(1)\times su(3))\rho_{L+}\rho_{R+}.$$

This is precisely what it seems.

The situation is more complicated than this (see [1][2]), but the overriding point being made here is that the mathematics of $\mathbf{T}$ can be viewed as implying we exist in an observable universe that must be dominantly matter, or anti-matter (if we accept that everything carrying nontrivial $SU(3)$ color charges is not directly observable by us, which in this context includes quarks, anti-quarks, and the extra 6 dimensions of spacetime, all of which involve the octonion units, $e_p,\ p = 1, 2, 3, 4, 5, 6$, which carry those charges). Acceptance of this notion has the potential to imply far more profound things about physics.

For example, in [1] it was pointed out that the original model allowed algebraically for matter-antimatter mixing via the extra 6 dimensions, but that reasonable conditions put on the dependence of the various particle fields on these extra dimensions led to these mixing pathways disappearing. Whatever the case, in the present context this idea of mixing needs to be rethought. The extra 6 dimensions provide channels from the matter universe to the antimatter universe. Were these channels viable they would allow, for example, an electron from our matter universe to channel through to the anitmatter universe, appearing on the other side as an antiquark (it necessarily picks up an anti-color charge en route). But this idea just scratches the surface.

A final comment: this exploitation of $\mathbf{T}$ as the foundation of a model of reality is not the only one, it is the one I like best (well, I've been at it for over 30 years, so changing now is not going to happen). For an alternate approach, see [5].

References

[1] G.M. Dixon, *Division Algebras: Octonions, Quaternions, Complex Numbers, and the Algebraic Design of Physics*, (Kluwer (Now Springer), 1994).

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[4] Pascual Jordan, John von Neumann, Eugene Wigner, *On an algebraic generalization of the quantum mechanical formalism*, Ann. Math. 35 (1934), 29-64.

[5] Cohl Furey, *Unified theory of ideals*, Phys. Rev. D 86 (2012).